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GRAVITOELECTRIC-ELECTRIC COUPLING VIA SUPERCONDUCTIVITY

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Recently we demonstrated theoretically that the carriers of quantized angular momentum in superconductors are not the Cooper pairs but the lattice ions, which must execute coherent localized motion consistent with the phenomenon of superconductivity. We demonstrate here that in the presence of an external magnetic field, the free superelectron and bound ion currents largely cancel providing a self-consistent microscopic and macroscopic interpretation of near-zero magnetic permeability inside superconductors. The neutral mass currents, however, do not cancel, because of the monopolar gravitational charge. It is shown that the coherent alignment of lattice ion spins will generate a detectable gravitomagnetic field, and in the presence of a time-dependent applied magnetic vector potential field, a detectable gravitoelectric field.

Key words: superconductivity, gravitation, gravitomagnetism.

1. INTRODUCTION

In the weak field low-velocity limit of general relativity, the field equations in free space can be written approximately in the form of Maxwell's equations[1-3]. We find that when applied to superconductors, however, the effects of the material properties of superconductors on the gravitational fields must be taken into account, and we do this by introducing gravitational analogs of the electric permittivity and magnetic permeability. With these changes, the field equations can be written in the form:

$$\nabla \cdot \mathbf{E}_g = -\frac{\rho_m}{\varepsilon_g}, \ \nabla \cdot \mathbf{B}_g = 0, \tag{1.1}$$

$$\nabla \times \mathbf{E}_g = -\frac{\partial \mathbf{B}_g}{\partial t}, \ \nabla \times \mathbf{B}_g = -S^2 \mu_g \mathbf{j}_m + \mu_{go} \varepsilon_g \frac{\partial \mathbf{E}_g}{\partial t}, \tag{1.2}$$

and, in terms of the London gauge potentials,

$$\mathbf{E}_g = -\nabla \phi_g - \frac{\partial \mathbf{A}_g}{\partial t}, \ \mathbf{B}_g = \nabla \times \mathbf{A}_g, \tag{1.3}$$

where $\mathbf{E}_g$ is referred to as the gravitoelectric field, $\mathbf{B}_g$ is called the gravitomagnetic field;[4] $\mathbf{j}_m$ and ρ_m are the free mass "charge" current and density, respectively; μ_g, ε_g and $\mu_{g,o}$, $\varepsilon_{g,o}$ are the gravito permeability and gravito permittivity and their free space values respectively (discussed later); and ϕ_g and $\mathbf{A}_g$ are the gravito scalar and London gauge potentials, respectively. The parameter S represents the helicity of the field which is not relevant to the objectives of this paper and is assumed to be equal to one. The equations of motion of a Cooper pair of mass m moving with the velocity $\mathbf{V}$ in electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$, respectively, and gravitoelectric and gravitomagnetic fields $\mathbf{E}_g$ and $\mathbf{B}_g$, respectively, can be written to a first approximation in terms of a generalized Lorentz force[1-3]:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{V} \times \mathbf{B}) + m(\mathbf{E}_g + \mathbf{V} \times \mathbf{B}_g). \tag{1.4}$$

At a higher level of approximation, additional terms will appear in Eq. (1.4) which arise from the spatial components of the metric in the PPN formalism, for which there is no electromagnetic counterpart. The similarity between Eqs. (1.1) to (1.3) and Maxwell's equations for the propagation of electromagnetic fields in material media is apparent, and this had led to the expectation, at least to a first-order approximation, that many of the phenomena which occur in electromagnetism should have gravitational counterparts which, by analogy, may be collectively grouped under the title "gravitoelectromagnetism."

The purpose of this paper is to investigate further the analogy between electromagnetism and gravitoelectromagnetism by studying the effects of electric and gravitoelectric coupling. Recently we[5] demonstrated that, in the case of a superconductor, an external magnetic field will induce an

internal gravitomagnetic field, the magnitude of which will depend on the value the ratio μ_g/μ assumes for superconductors. Because the free-space value of μ_g/μ, denoted by μ_{go}/μ_o (where $\mu_{go} = \frac{1}{\varepsilon_{go}c^2}$ and $\varepsilon_{go} = \frac{1}{4\pi G}$, G denoting the gravitation constant and, c the speed of light) is very small (7.4×10^{-21} $coul^2/kg^2$), it is clear that in free space any induced gravitational fields will be very small. We, therefore, focus on fields in superconductors, where μ_g/μ has been demonstrated[6] to be significantly larger than μ_{go}/μ_o. In this paper we provide a theoretical basis for a measurable electrically induced gravitoelectric field due to a large increase in μ_g/μ, which arises mainly from the near-zero magnetic permeability observed for some superconductors. The specific topic we address here is establishing a principle for the electrical induction of potentially measurable gravitational fields in an ideal superconductor.

2. THE INDUCED GRAVITOELECTRIC (GRAVITATIONAL) FIELD

In our approach we treat the ideal superconductor as a many-particle system such that, in the presence of the applied electromagnetic and gravitoelectromagnetic potentials $\mathbf{A}$, Φ, $\mathbf{A}_g$, Φ_g, the Langrangian can be written as

$$L = \sum_j \frac{1}{2} m_j v_j^2 - \sum_j q_j(\Phi - \mathbf{v}_j \cdot \mathbf{A}) - \sum_j m_j(\Phi_g - \mathbf{v}_j \cdot \mathbf{A}_g). \qquad (2.1)$$

The canonical momentum for the jth particle,

$$-i\hbar\nabla_j = m_j\mathbf{v}_j + q_j\mathbf{A} + m_j\mathbf{A}_g, \qquad (2.2)$$

computed from the Lagrangian, yields the macroscopic averaged *total* electric current density $\mathbf{j}_e$ and the mass current density $\mathbf{j}_m$, which in conjunction with the Ginsburg-Landau equations for the Cooper pair distribution function, can be written as[7]

$$\mathbf{j}_e = \sum_j \frac{q_j \hbar}{2im_j}(\psi^*\nabla_j\psi - \psi\nabla_j\psi^*) - \sum_j \frac{q_j^2}{m_j}(\mathbf{A} + \frac{m_j}{q_j}\mathbf{A}_g)\psi^*\psi, \qquad (2.3)$$

$$\mathbf{j}_m = \sum_j \frac{\hbar}{2i}(\psi^*\nabla_j\psi - \psi\nabla_j\psi^*) - \sum_j q_j(\mathbf{A} + \frac{m_j}{q_j}\mathbf{A}_g)\psi^*\psi. \qquad (2.4)$$

Equations (2.3) and (2.4) are the usual quantum mechanical equations of the charge and mass current densities for a many-particle system, where each particle has a charge q_j and mass m_j. $\psi = |\psi|e^{i\phi}$ is the superconducting order parameter with phase ϕ. For a superconductor one need only consider two kinds of particles, namely the Cooper pairs and the lattice ions. The wave function ψ forms a many-particle condensate coherent wave, such that the local density of the superconducting electrons is given by $n_s \approx \psi\psi^*$.

We can therefore identify the "free" currents with the vector potentials **A** and $\mathbf{A_g}$, i.e.,

$$\mathbf{j}_{ef} = -Q(\mathbf{A} + \frac{m}{q}\mathbf{A}_g), \qquad (2.5)$$

$$\mathbf{j}_{mf} = -Q\frac{m}{q}(\mathbf{A} + \frac{m}{q}\mathbf{A}_g), \qquad (2.6)$$

where

$$Q = \frac{q^2}{m}|\psi|^2 \qquad (2.7)$$

is the kernel function.

If the Cooper pairs comprising the superconducting current carriers are formed by bound pairs of electrons occupying states with equal and opposite momentum and spin[7], i.e. s-state pairing, they are characterized by zero total angular momentum and spin. The bound currents, which we identify as the sources of the magnetizations **M** and **L** (the macroscopically averaged angular momentum density), are given by

$$\mathbf{j}_{eM} = \nabla \times \mathbf{M} = \frac{q_i\hbar}{2im_i}(\psi^*\nabla_i\psi - \psi\nabla_i\psi^*) - Q_i(\mathbf{A} + \frac{m_i}{q_i}\mathbf{A}_g)$$

$$= \frac{q_i|\psi|^2 2\hbar\nabla_i\phi}{2m_i} - Q_i(\mathbf{A} + \frac{m_i}{q_i}\mathbf{A}_g) = 2M_i|\psi|^2\nabla_i\phi - Q_i(\mathbf{A} + \frac{m_i}{q_i}\mathbf{A}_g), \qquad (2.8)$$

$$\mathbf{j}_{mM} = \frac{1}{2}\nabla \times \mathbf{L} = \frac{\hbar}{2i}(\psi^*\nabla_i\psi - \psi\nabla_i\psi^*) - Q_i\frac{m_i}{q_i}(\mathbf{A} + \frac{m_i}{q_i}\mathbf{A}_g)$$

$$= |\psi|^2\, 2\hbar\nabla_i\phi - Q_i\frac{m_i}{q_i}(\mathbf{A} + \frac{m_i}{q_i}\mathbf{A}_g) = |\psi|^2\mathbf{p}_i - Q_i\frac{m_i}{q_i}(\mathbf{A} + \frac{m_i}{q_i}\mathbf{A}_g), \qquad (2.9)$$

where $Q_i = \frac{q_i^2}{m_i}|\psi|^2$; the subscript M represents the fact that these are bound currents; q_i and m_i designate the charge and mass of the lattice ions respectively, and ∇_i acts only on ions. The quantity $M_i = q_i\hbar/2m_i$ is identified as the Bohr magneton of a lattice ion. In order to maintain charge neutrality, the density of the lattice ions is assumed equal to that of the Coopers pairs, i.e., $|\psi|^2/2$. Integrating the ion canonical momentum $\mathbf{p}_i$ given by $\mathbf{p}_i = 2\hbar\nabla_i\phi$ around a closed path, one finds that[8]

$$\oint \mathbf{p}_i \cdot d\mathbf{l} = 2nh. \qquad (2.10)$$

Equation (2.10) with n >1 (see Ref. [6]) provides a basis for the macroscopic quantum property of superconductivity and identifies the lattice ions as the carriers of quantized angular momentum. The ion current arises from the interaction energy between the magnetic dipole moment $\mu_i = M_i L_i$ of a superlattice ion and an effective quantum vector potential $A_q = \Phi_o\nabla_i\phi$ generated by the phase coherence of the superconducting condensate wave function $\psi = |\psi|e^{i\phi}$, where $\Phi_o = \frac{\hbar}{q}$ is the fluxoid quantum; $\hbar L_i$ is the quantum angular momentum plus spin of a superlattice ion. In other words, the lattice ions cannot possess zero quantized angular momentum, and therefore must possess sufficient freedom of motion in the form of localized vortical motion or spin within the vicinity of the superlattice nodes. The quantum vector potential A_q that arises from the phase coherence of the superconducting condensate wave function requires intrinsic coherent behavior from the current carrier.

Using Eqs. (2.8) and (2.9) with (2.5) and (2.6), we obtain

$$\mu = \mu_o\,(1 + \frac{m}{m_i} + \frac{\mathbf{j}_{eM}\cdot\mathbf{j}_{ef}}{j_{ef}^2}) = \mu_r\,\mu_o\,, \qquad (2.11)$$

$$\mu_g = \mu_{go}\,(1+\frac{m}{m_i}+\frac{\mathbf{j}_{mM}\cdot\mathbf{j}_{mf}}{j_{mf}^2}) = \mu_{gr}\,\mu_{go}. \tag{2.12}$$

Note that the term m/m_i takes account of the vector potential term $(\mathbf{A}+\frac{m_i}{q_i}\mathbf{A}_g)$ in Eqs. 2.8 and 2.9, if the small $\mathbf{A}_g$ component is neglected (for justification see Eq. 3.7, for example). Note also, in Eqs. (2.11) and (2.12) the product $\mathbf{j}_{eM}\cdot\mathbf{j}_{ef}$ is negative, since the electrical current $\mathbf{j}_{eM}$ is carried by the ions, and $\mathbf{j}_{ef}$ is carried by Cooper pairs, whereas $\mathbf{j}_{mM}\cdot\mathbf{j}_{mf}$ is positive, since the gravitational charge (mass) is monopolar. This results in a reduction in μ_r and an increase in μ_{gr}. Later we present evidence which suggests that inside the superconductor $|\mathbf{j}_{eM}| \approx |\mathbf{j}_{ef}|$. It can be easily shown from Eqs. (26) and (27) in Ref. 5 that boundary conditions require that μ increase in the screen layer to the point that the internal field generated largely cancels the external magnetic field, $\mathbf{H}_o$. It should also be noted, however, that the spatially variable μ does not introduce a $\nabla \times \mu$ term when the curl is taken of Eq. 3 in Ref. 5 ($\nabla \times \mathbf{B} = \mu \mathbf{j}_{ef}$) since μ is a scalar; also, the additional term $\frac{nq^2}{m}\,\mathbf{A} \times \nabla\mu$ is assumed to be zero and will be considered in a later paper.

Equations (2.5) and (2.6) yield the time-dependent supercurrents

$$\frac{\partial \mathbf{j}_{ef}}{\partial t} = \frac{-1}{\mu\lambda_L^2}\left(\frac{\partial \mathbf{A}}{\partial t}+\frac{m}{q}\frac{\partial \mathbf{A}_g}{\partial t}\right), \tag{2.13}$$

$$\frac{\partial \mathbf{j}_{mf}}{\partial t} = \frac{-1}{\mu\lambda_L^2}\left(\frac{m}{q}\frac{\partial \mathbf{A}}{\partial t}+\frac{m^2}{q^2}\frac{\partial \mathbf{A}_g}{\partial t}\right). \tag{2.14}$$

Substituting for $\partial \mathbf{j}_{ef}/\partial t$ in the equation

$$\nabla^2\mathbf{E} = \mu\frac{\partial \mathbf{j}_{ef}}{\partial t} + \mu_o\varepsilon\,\frac{\partial^2 \mathbf{E}}{\partial t^2} + \frac{1}{\varepsilon}\,\nabla\rho_e \tag{2.15}$$

and for $\partial \mathbf{j}_{mf}/\partial t$ in the analogous gravitolectric counterpart

$$\nabla^2 \mathbf{E}_g = -\mu_g \frac{\partial \mathbf{j}_{mf}}{\partial t} + \mu_{go}\varepsilon_g \frac{\partial^2 \mathbf{E}_g}{\partial t^2} - \frac{1}{\varepsilon_g}\nabla \rho_m, \tag{2.16}$$

where ρ_e and ρ_m are free charge and mass densities, respectively, we obtain

$$\nabla^2 \mathbf{E} = \frac{1}{\lambda_L^2}\left[\mathbf{E} + \frac{m}{q}\mathbf{E}_g + \nabla(\phi + \frac{m}{q}\phi_g)\right] + \mu_o\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} + \frac{1}{\varepsilon}\nabla \rho_e\,, \tag{2.17}$$

$$\nabla^2 \mathbf{E}_g = -\frac{1}{\lambda_L^2}\frac{\mu_g}{\mu}\frac{m}{q}\left[\mathbf{E} + \frac{m}{q}\mathbf{E}_g + \nabla(\phi + \frac{m}{q}\phi_g)\right] + \mu_{go}\varepsilon_g \frac{\partial^2 \mathbf{E}_g}{\partial t^2} - \frac{1}{\varepsilon_g}\nabla \rho_m. \tag{2.18}$$

In order to obtain analytic solutions of Eqs. (2.17) and (2.18), we consider the special case of a half-infinite superconductor with time-dependent external electric and gravitational vector potentials parallel to the superconducting surface, i.e.,

$$\mathbf{E}_o = (0, \frac{\partial A_o}{\partial t}, 0), \quad \mathbf{E}_{go} = (0, \frac{\partial A_{g\,o}}{\partial t}, 0). \tag{2.19}$$

The boundary conditions that must be applied to the solutions of Eqs. (2.17) and (2.18) are

$$\mathbf{E}_{z,int} - \mathbf{E}_{z,ext} = \sigma_E/\varepsilon\,, \quad \mathbf{E}_{gz,int} - \mathbf{E}_{gz,ext} = -\sigma_m/\varepsilon_g, \tag{2.20}$$

$$\mathbf{E}_{y,int} - \mathbf{E}_{y,ext} = 0, \quad \mathbf{E}_{gy,int} - \mathbf{E}_{gy,ext} = 0, \tag{2.21}$$

where σ_e and σ_m are the surface electric and mass charge densities respectively and subscripts "int" and "ext" refer to internal and external fields respectively. We note that for constant applied fields the time dependent current terms in Eqs. (2.2) and (2.3) are not zero, but constants. For convenience, we assume $\partial^2/\partial t^2 = 0$ without loss of the time-dependent vector potential source. As the neglected terms arise from the terms $\partial^2/\partial t^2$ in the fundamental Eqs. (2.17) and (2.18), it means we are neglecting the change of the displacement currents as compared with that of the conduction currents. In the case of the half-infinite superconductor, there are no boundaries at which either electric or mass charge can accumulate to generate a potential difference. We assume that

the external fields are uniform. Since the London gauge potential satisfies the condition $\nabla \cdot \mathbf{A} = 0$; $\nabla \cdot \mathbf{A}_g = 0$, the Maxwell equations $\nabla \cdot \mathbf{E} = \rho_e/\varepsilon$, $\nabla \cdot \mathbf{E}_g = -\rho_m/\varepsilon_g$ can be written as $\nabla^2\phi = -\rho_e/\varepsilon$; $\nabla^2\phi_g = \rho_m/\varepsilon_g$. Therefore, Eqs. (2.17) and (2.18) reduce to

$$\nabla^2(\mathbf{E} + \nabla\phi) = \frac{1}{\lambda_L^2}\left[(\mathbf{E} + \nabla\phi) + \frac{m}{q}(\mathbf{E}_g + \nabla\phi_g)\right], \tag{2.22}$$

$$\nabla^2(\mathbf{E}_g + \nabla\phi_g) = -\frac{1}{\lambda_L^2}\frac{m}{q}\frac{\mu_g}{\mu}\left[(\mathbf{E} + \nabla\phi) + \frac{m}{q}(\mathbf{E}_g + \nabla\phi_g)\right]. \tag{2.23}$$

Setting $\mathbf{E} + \nabla\phi = -\dfrac{\partial \mathbf{A}}{\partial t} = \tilde{\mathbf{E}}$ and $\mathbf{E}_g + \nabla\phi_g = -\dfrac{\partial \mathbf{A}_g}{\partial t} = \tilde{\mathbf{E}}_g$ (i. e., retaining only the component due to the vector potential) and imposing the boundary conditions yield the following solutions for $\mathbf{E}$ and $\mathbf{E}_g$:

$$\frac{\partial \mathbf{A}}{\partial t} = \left[1 - \frac{\mu_g}{\mu}\frac{m^2}{q^2}\right]^{-1}\left[\frac{\partial \mathbf{A}_o}{\partial t}\left(-\frac{\mu_g}{\mu}\frac{m^2}{q^2} + e^{-z/\lambda}\right) - \frac{\partial \mathbf{A}_{go}}{\partial t}\frac{m}{q}(1 - e^{-z/\lambda})\right], \tag{2.24}$$

$$\frac{\partial \mathbf{A}_g}{\partial t} = \left[1 - \frac{\mu_g}{\mu}\frac{m^2}{q^2}\right]^{-1}\left[\frac{\partial \mathbf{A}_{go}}{\partial t}\left(1 - \frac{\mu_g}{\mu}\frac{m^2}{q^2} + e^{-z/\lambda}\right) + \frac{\partial \mathbf{A}_o}{\partial t}\frac{\mu_g}{\mu}\frac{m}{q}(1 - e^{-z/\lambda})\right], \tag{2.25}$$

where $\lambda^2 = \lambda_L^2[1 - (\mu_g/\mu)(m^2/q^2]^{-1}$. Equations (2.24) and (2.25) yield the result

$$\tilde{\mathbf{E}} + \frac{m}{q}\tilde{\mathbf{E}}_g = \left(\frac{\partial \mathbf{A}_o}{\partial t} + \frac{m}{q}\frac{\partial \mathbf{A}_{go}}{\partial t}\right)e^{-z/\lambda}. \tag{2.26}$$

Within a superconductor, $z \gg \lambda$, and we obtain[9]

$$\tilde{\mathbf{E}} + \frac{m}{q}\tilde{\mathbf{E}}_g = 0, \tag{2.27}$$

which indicates that the internal volume of a superconductor is a region of force-free fields. (Note that this condition only applies to idealized superconductors in which there are no trapped fields.) More importantly, we find that constant residual fields exist inside a superconductor, which are given by

$$\tilde{E}(z) = \left[1 - \frac{\mu_g}{\mu}\frac{m^2}{q^2}\right]^{-1}\left[-\frac{\mu_g}{\mu}\frac{m^2}{q^2}\frac{\partial A_o}{\partial t} - \frac{m}{q}\frac{\partial A_{go}}{\partial t}\right] \approx -\beta\tilde{E}_o - \frac{m}{q}\tilde{E}_{go}, \tag{2.28}$$

$$\tilde{E}_g(z) = \left[1 - \frac{\mu_g}{\mu}\frac{m^2}{q^2}\right]^{-1}\left[\frac{\partial A_{go}}{\partial t} + \frac{\mu_g}{\mu}\frac{m}{q}\frac{\partial A_o}{\partial t}\right] = \beta\frac{q}{m}\tilde{E}_o + \tilde{E}_{go}, \tag{2.29}$$

where

$$\beta = \frac{\mu_g}{\mu}\frac{m^2}{q^2} \tag{2.30}$$

is the screening factor or the electric attenuation coefficient, which is assumed to be much smaller than unity. For example, Schiff's screening factor[10] which is of the order of 10^{-7} is used in the Stanford ^{3}He Nuclear Gyroscope experiment[11]. Equation (2.29) indicates that the internal $\tilde{E}_g$ field consists of two terms: the external $\partial A_{go}/\partial t$ field plus an induced $\tilde{E}_g$ field generated by $\partial A_o/\partial t$. Thus, even when $\partial A_{go}/\partial t = 0$, there will be an electrically induced gravitoelectric field given by

$$\tilde{E}_g^{\,ind} = \frac{\mu_g m}{\mu q}\tilde{E}_o. \tag{2.31}$$

Whether or not this $\tilde{E}_g^{\,ind}$ field will be detectable will depend on the value $\frac{\mu_g}{\mu}$ assumes for superconductors.

3. EVALUATION OF THE $\tilde{E}_g^{\,ind}$ FIELD

Using Eqs. (2.11), (2.12), with (2.5), (2.6), (2.8), and (2.9) and the experimental fact that $\mu_r << 1$, we obtain

$$\mu_g = \frac{\gamma_e \mu_{go}}{\gamma_i} \approx 10^4 \mu_{go}, \tag{3.1}$$

where $\gamma_i = q_i/2m_i$ and $\gamma_e = q/2m$ is the gyromagnetic ratio for a Cooper pair, and it is assumed that $\frac{\gamma_e}{\gamma_i} \approx \frac{m_i}{m} \approx 10^4$ for superconductors. Then, we use Eq. (31) in Ref. [4] (with $\mathbf{B}_{go} = 0$) and experimental results to

determine μ. Equation (31) in Ref. [5] shows that an internal residual magnetic field

$$B = -\frac{\mu_g}{\mu}\frac{m^2}{q^2} B_o \tag{3.2}$$

will exist within a superconductor and is also characterized by the same attenuation coefficient β. Equation (3.2) can be written in the alternate form

$$H = -\frac{\mu_g \mu_o}{\mu^2}\frac{m^2}{q^2} H_o. \tag{3.3}$$

Equations (3.2) and (3.3) yield an internal magnetization such that

$$M = -\frac{\mu_g}{\mu}\frac{m^2}{q^2}\left(\frac{\mu_o}{\mu} - 1\right) H_o, \tag{3.4}$$

since $\mathbf{B} = \mu_o (\mathbf{H} + \mathbf{M})$. The physical picture can be briefly described as follows. In the presence of a external magnetic field, the gradient of the phase of the superconducting condensate wavefunction within a superconductor will induce a small internal magnetic field[12] or the magnetization as shown by Eq. (3.4), the interaction of which with the magneton of the lattice ions gives rise to a persistent bound vortical current in the range of the coherent length as illustrated by Eq. (2.8). If one believes that this persistent vortical bound current can be represented by the persistent current observed in small normal metal rings, we may use the new experimental observations recently published by Bell Labs[13] and IBM[14] independently to calculate the magnetic moment density. The experimental results of Bell Labs and IBM show that in the presence of a magnetic field and in a sufficiently small ring of 0.5×10^{-6} m in diameter, there is a persistent current of order 0.1 nA with an associated induced very small magnetic moment, which incidentally suggests that normal metals become conductors with infinite conductivity in sufficiently small volumes. The net magnetic moment density can be calculated from

$$\mathbf{M} = N \pi a^2 I \mathbf{n}, \tag{3.5}$$

where $N = 1/(\frac{4\pi}{3} a^3)$ is the density, πa^2 is the area, I the current, **n** is the normal direction of the small area, and a is the radius. Substituting these experimental results into Eqs. (3.5) and (3.4), we obtain

$$\mu_r \approx 10^{-17}, \tag{3.6}$$

where we have assumed that the external magnetic field is the earth's magnetic field $B_{earth} = 5 \times 10^{-5}$ tesla and $\frac{m_i}{m} \approx 10^4$ for superconductors. The experimentally inferred values for μ_r based on the small metal results given above agree reasonably well with our theoretical results[6] of $\mu_r \approx 10^{-20}$ for superconductors, suggesting a possible importance of gravitational effects for the residual internal fields in superconductors. More important, it appears that the electrically induced gravitoelectric field

$$\tilde{E}_g^{ind} \approx 10^{-9} \frac{\partial A_o}{\partial t} \quad (\mathrm{ms}^{-2}) \tag{3.7}$$

should be readily detectable in the laboratory.

In the presence of a constant external **B** field (i.e., $\partial A_o/\partial t = 0$), only the magnetic-like terms survive, and the internal neutral currents generate a gravitomagnetic field given by

$$\mathbf{B}_g = \frac{\mu_g}{\mu} \frac{m}{q} \mathbf{B}_o. \tag{3.8}$$

The corresponding value of this field is predicted to be greater than $\sim 10^{-9}$ B_o, which is six orders of magnitude larger than that of the earth at ~ 650 km altitude for B_o = 1tesla. It is easy to demonstrate that this is an intuitively reasonable result . Using the standard expression used for computing B_g for the earth, namely

$$\mathbf{B}_{g,earth} = \frac{\mu_{g,o}}{4\pi} \frac{\mathbf{I}\omega}{R^3} \tag{3.9}$$

to compute the gravitomagnetic field generated by a single lattice ion, where I is the moment of inertia, ω the angular velocity, and R the distance to the observer, we find

$$B_{g,\mathrm{ion}} \approx 10^{-37}\ \mathrm{Hz} \tag{3.10}$$

per ion. Taking an arbitrary volume of $0.1\mathrm{m}^3$, there are approximately 10^{28} superlattice ions executing perfectly coherent vortical rotation. Using dimensions consistent with laboratory scales, and assuming spin angular momentum only, we find the total B_g field generated is ~ 10^{-9} Hz. The above calculation is straightforward and demonstrates that there is no difficulty in understanding the mechanism operative.

When the external magnetic vector potential is ramped up, a time dependent gravitomagnetic vector potential is generated which is the E_g field. Clearly because of the coupling of the gravitational and electric charges through a common carrier, the vector magnetic potential plays the role of a vector gravitomagnetic potential. This becomes self-evident when Eqs. (2.31) or (3.7) are written in the form

$$\widetilde{\mathbf{E}}_g = \frac{-\partial \mathbf{A}_g}{\partial t} = \frac{\mu_g}{\mu}\frac{m}{q}\frac{\partial \mathbf{A}_o}{\partial t}. \tag{3.11}$$

The effective time-dependent gravitomagnetic potential becomes very large because of the inverse dependence on μ which is very small. The small value for μ is caused by the fact that the internal quantized electrical ion current opposes that generated by the non-quantized Cooper pair free current resulting in a net small current which results in the condition $\mu_r \approx 0$ in Eq. (2.11). In Eq. (2.12) j_{mM} and j_{mf} are of equal sign so no current cancellation occurs, yielding $\mu_{gr} = 10^4$. The fact that the mass of the carrier of the j_{mM} current in Eq. (2.12) is that of the ions, whereas the j_{mf} current is carried by Cooper pairs, results in the factor of 10^4 effective increase in μ_{gr}.

The final value assumed by μ_r under equilibrium conditions is determined by the requirement given by Eq. (2.27), namely that the superconductor must remain a force free region. The net result is the generation of potentially detectable gravitoelectric and gravitomagnetic fields. Experimental validation of the theoretical predication would verify the gravitational analogs of Maxwell's equations and further confirm general relativity.

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